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Instructional Routines for Mathematics Intervention

MODULE 2



Module 2: Comparison

Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
compare	To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.
denominator	The term in a fraction that tells the number of equal parts in a whole.
digit	A symbol used to show numbers.
equal	When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
greater than	When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.
hundreds	The digit representing 100.
less than	When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.
number line	A straight line with numbers placed at equal intervals along its length.
numerator	The term in a fraction that tells how many parts in a fraction.
ones	The digit representing 1.
place value	The value of a digit depending on its place in a number.
rational number	Any number that can be written as a fraction.
tens	The digit representing 10.
thousands	The digit representing 1,000.




B. Background Information

Comparison is important for students to understand numbers as greater, less, or equal.

Typically, students first learn to compare (1) whole numbers. Then, students learn to compare

(2) fractions and decimals. Decimals can be compared using the same strategy as comparing whole numbers, so we provide an overview of both in section (1).

When teaching about comparison, emphasize place value. Also, emphasize vocabulary related to comparison, such as *greater than*, *less than*, *equal to*, and *equivalent*, and the symbols representing this vocabulary.

Comparison		
$16 > 9$	$1.3 < 1.35$	$68 = 68$
		
greater than symbol	less than symbol	equal sign

C. Routines and Examples

(1) Comparing Whole Numbers and Decimals

Routine

Materials:

- [Module 2 Problems](#)
- [Module 2 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., clips, Base-10 blocks)

Teacher Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?

Students To determine whether a number is greater than, less than, or equal to another number.

Teacher Today, we'll compare numbers with these Base-10 blocks.
(Show Base-10 blocks.)

Teacher With Base-10 blocks, one cube represents one thousand. What does a cube represent?

Students One thousand.

Teacher The flat represents one hundred. What does the flat represent?

Students One hundred.

Teacher The rod represents one ten. What does the rod represent?

Students One ten.

Teacher **And the unit represents one. What does the unit represent?**

Students One.

Teacher **Now, let's compare numbers. Let's compare ___ and ___. What numbers are we going to compare?**

Students ___ and ___.

Teacher **And for this comparison, we want to determine if ___ (first number) is greater than, less than, or equal to ___ (second number). What do we want to do?**

Students Determine if the first number is greater than, less than, or equal to the second number.

Teacher **Now, let's compare numbers. Let's make the first number with the Base-10 blocks. How could I show ___?**

Students You could use ___.

Teacher **I'll show ___ (first number) by showing ___.**
(Show using Base-10 blocks.)

Teacher **Let's make the second number with Base-10 blocks. I'll place my blocks over here (on other side of workspace). How could I show ___?**

Students You could use ___.

Teacher **I'll show ___ (second number) by showing ___.**
(Show using Base-10 blocks.)

Teacher **Now, it's time to compare. Look at the greatest place value. What's the greatest place value?**

Students ___.

Teacher **___ is the greatest place of ___ (first number) and ___ (second number). Look at the first number, how many ___ (greatest place value)?**

Students ___.

Teacher **Look at the second number, how many ___ (greatest place value)?**

Students ___.

Teacher **Are the ___ (greatest place value) of the first number the same or different from ___ (greatest place value) of the second number?**

Students *OPTION 1:* The same!
OPTION 2: Different.

Teacher ***OPTION 1:* When the greatest place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?**

Students ___.

Teacher **That's right. The next greatest place value is the ___ place. Look at the first number, how many ___ (place value)?**

Students ___.

Teacher **Look at the second number, how many ___ (place value)?**

Students ___.

Teacher **Are the ___ (place value) of the first number the same or different from ___ (greatest place value) of the second number?**

Students *OPTION 1:* The same!

Teacher *OPTION 2:* Different.
OPTION 1: When the place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?

Students _____.

Teacher That's right. The next greatest place value is the ____ place. Look at the first number, how many ____ (place value)?

Students _____.

Teacher Look at the second number, how many ____ (place value)?

Students _____.

Teacher Are the ____ (place value) of the first number the same or different from ____ (greatest place value) of the second number?

Students *OPTION 1:* The same!
OPTION 2: Different.

Teacher **OPTION 2:** The ____ (place value) of the first number is different from the ____ (place value) of the second number. If the digits are different, then we can compare. What can we do?

Students Compare.

Teacher Is the ____ (place value) of the first number greater than, less than, or equal to that of the second number?

Students _____.

Teacher If it's greater, that means ____ (first number) is greater than ____ (second number). If it's less, that means ____ (first number) is less than ____ (second number). If the numbers are the same, they are equal. What's the comparison?

Students ____ (greater/less/equal).

Teacher That's right! ____ (first number) is ____ (greater than/less than/equal to) ____ (second number). Let's say that together.

Students ____ is greater than/less than/equal to _____.

Teacher Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?

Students _____.

Teacher Let's write the symbol between the two numbers.
 (Write.)

Teacher What does it mean to compare numbers?

Students We determine whether one number is greater than, less than, or equal to another number.

Teacher How did we compare numbers in this example?

Students We compared each digit by place value then determined whether one number was greater than, less than, or equal to the other number.

Example

$$105.6 < 106.5$$

Teacher Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?

Students To determine whether a number is greater than, less than, or equal to another number.

Teacher Now, let's compare numbers. Let's compare 105.6 and 106.5. What numbers are we going to compare?

Students 105.6 and 106.5.

Teacher And for this comparison, we want to determine if 105.6 is greater than, less than, or equal to 106.5. What do we want to do?

Students Determine if the first number is greater than, less than, or equal to the second number.

Teacher Let's compare. Look at the greatest place value of the numbers. What's the greatest place value?

Students Hundreds.

Teacher Hundreds is the greatest place value of the numbers 105.6 and 106.5. Look at the first number, how many hundreds?

Students 1 hundred.

Teacher Look at the second number, how many hundreds?

Students 1 hundred.

Teacher Are the hundreds of the first number the same or different compared to the hundreds of the second number?

Students Equal or the same.

Teacher When the greatest place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?

Students Tens.

Teacher That's right. The next greatest place value is the tens place. Look at the first number, how many tens?

Students 0 tens.

Teacher Look at the second number, how many tens?

Students 0 tens.

Teacher Are the tens of the first number the same or different compared to the tens of the second number?

Students Equal or the same.

Teacher When the place value is the same, we look at the next greatest place value. I move one place value to the right. What's the next greatest place value?

Students Ones.

Teacher That's right. The next greatest place value is the ones place. Look at the first number, how many ones?

Students 5 ones.

Teacher Look at the second number, how many ones?

Students 6 ones.

Teacher Are the ones of the first number the same or different compared to the ones of the second number?

Students Different.

Teacher The ones of the first number are different from the ones of the second number. If the digits are different, then we can compare. What can we do?

Students Compare.

Teacher Let's compare. Are the ones of the first number greater than, less than, or equal to that of the second number?

Students Less.

Teacher It's less so that means 105.6 is less than 106.5. What's the comparison?

Students Less than.

Teacher That's right! 105.6 is less than 106.5. Let's say that together.

Students 105.6 is less than 106.5.

Teacher Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?

Students Less than symbol.

Teacher Let's write the less than symbol between the two numbers.
(Write.)

Teacher Let's read it together.

Students 105.6 is less than 106.5.

Teacher What does it mean to compare numbers?

Students To determine whether one number is greater than, less than, or equal to another number.

(2) Comparing Fractions*

*For clarity, read [Example](#) before using [Routines](#).

Routine

Materials:

- [Module 2 Problems](#)
- [Module 2 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., fraction tiles, geoboards)

Teacher Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?

Students To determine whether one number is greater than, less than, or equal to another number.

Teacher Today, we'll compare numbers with these fraction tiles.
(Show fraction tiles.)

Teacher Now, let's compare numbers. Let's compare ___ and ___. What numbers are we going to compare?

Students ___ and ___.

Teacher And for this comparison, we want to determine if ___ (first number) is greater than, less than, or equal to ___ (second number). What do we want to do?

Students Determine if the first number is greater than, less than, or equal to the second number.

Teacher Now, let's compare numbers. Let's make the first number with the fraction tiles. How could I show ___?

Students You could use ___.

Teacher I'll show ___ (first number) by showing ___. Remember, I want to show the fraction compared to the whole.

(Show using fraction tiles.)

Teacher Let's make the second number with fraction tiles. I'll place my fraction tiles over here (on other side of workspace). How could I show ___?

Students You could use ___.

Teacher I'll show ___ (second number) by showing ___. Remember, I want to show the fraction compared to the whole.

(Show using fraction tiles.)

Teacher Now, it's time to compare. What are we going to do?

Students Compare.

Teacher Let's think about the value of each fraction compared to the whole. Let's place both fractions on top of the whole to compare.

(Place fractions compared to whole.)

Teacher Look at the first number, is this fraction less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?

Students ___.

Teacher The first number is ___ than $\frac{1}{2}$. Let's remember that. Look at the second number, is this fraction less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?

Students ___.

Teacher The second number is ___ than $\frac{1}{2}$. Let's remember that. Now, if one fraction is less than or equal to $\frac{1}{2}$ and the other fraction is greater than or equal to $\frac{1}{2}$, then it's easy to compare. Is one fraction less than $\frac{1}{2}$ and the other greater than $\frac{1}{2}$?

Students *OPTION 1:* Yes. (Skip Option 2.)

OPTION 2: No.

Teacher *OPTION 2:* If both fractions are less than $\frac{1}{2}$ or greater than $\frac{1}{2}$, then we have to look at the value of each fraction a little closer. Is one fraction greater in length or area than the other fraction?

Students Yes.

Teacher What do you notice about ___ (first fraction) compared to ___ (second fraction)?

Students ___.

Teacher So, we can see that the value of the first fraction is different from the value of the second fraction.

Teacher It's time to compare. What should we do?

Students Compare.

Teacher Is the ___ (first fraction) greater than, less than, or equal to that of the second fraction?

Students ___.

Teacher If it's greater, that means ___ (first number) is greater than ___ (second number). If it's less, that means ___ (first number) is less than ___ (second number). If the numbers are the same, they are equal. What's the comparison?

Students ___ (greater than/less than/equal to).

Teacher That's right! ___ (first number) is ___ (greater than/less than/equal to) ___ (second number). Let's say that together.

Students ___ is greater/less/equal to ___.

Teacher Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?

Students ___.

Teacher Let's write the symbol between the two numbers.
(Write.)

Teacher What does it mean to compare numbers?

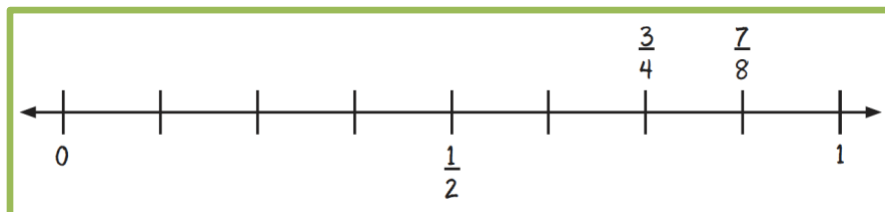
Students To determine if one number is greater than, less than, or equal to another number.

Teacher How did we compare numbers in this example?

Students We compared each fraction and then determined whether one number was greater than, less than, or equal to the other number.

Example

$$\frac{7}{8} > \frac{3}{4}$$



Teacher **Let's work on comparing numbers. Comparing means to determine whether a number is greater than, less than, or equal to another number. What does comparing mean?**

Students To determine whether a number is greater than, less than, or equal to another number.

Teacher **Today, we'll compare numbers with this number line.**
(Show number line.)

Teacher **Before we place fractions on the number line, let's draw a number line. I'll mark this number line with 0, $\frac{1}{2}$, and 1. How will I mark the number line?**

Students With 0, $\frac{1}{2}$, and 1.

Teacher **Now, let's compare numbers. Let's compare $\frac{7}{8}$ and $\frac{3}{4}$. What numbers are we going to compare?**

Students $\frac{7}{8}$ and $\frac{3}{4}$.

Teacher **And for this comparison, we want to determine if $\frac{7}{8}$ is greater than, less than, or equal to $\frac{3}{4}$. What do we want to do?**

Students Determine if the first number is greater than, less than, or equal to the second number.

Teacher **Now, let's compare numbers. Let's draw the first number on a number line. How could I show $\frac{7}{8}$?**

Students You could make 8 equal parts and mark $\frac{7}{8}$ above the seventh one-eighth mark.

Teacher **I'll show $\frac{7}{8}$ by dividing the number line into 8 equal parts. Then, I'll write $\frac{7}{8}$ above the seventh equal part.**
(Draw and write.)

Teacher **Let's draw the second number on the same number line. How could I show $\frac{3}{4}$?**

Students You could make 4 equal parts and mark $\frac{3}{4}$ above the third one-fourth mark.

Teacher I'll show $\frac{3}{4}$ by dividing the number line into 4 equal parts. Then, I'll write $\frac{3}{4}$ above the third equal part.
(Draw and write.)

Teacher Now, it's time to compare. What are we going to do?

Students Compare.

Teacher Let's think about the value of each fraction compared to the whole. Look at the first number, is $\frac{7}{8}$ less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?

Students Greater than.

Teacher The first number is greater than $\frac{1}{2}$. Let's remember that. Look at the second number, is $\frac{3}{4}$ less than $\frac{1}{2}$ or greater than $\frac{1}{2}$?

Students Greater than.

Teacher The second number is greater than $\frac{1}{2}$. Let's remember that. Now, if one fraction is less than or equal to $\frac{1}{2}$ and the other fraction is greater than or equal to $\frac{1}{2}$, then it's easy to compare. Is one fraction less than $\frac{1}{2}$ and the other greater than $\frac{1}{2}$?

Students No.

Teacher If both fractions are less than $\frac{1}{2}$ or greater than $\frac{1}{2}$, then we have to look at the value of each fraction a little closer. Is one fraction greater in length or area than the other fraction?

Students Yes.

Teacher What do you notice about $\frac{7}{8}$ compared to $\frac{3}{4}$?

Students $\frac{7}{8}$ is greater in value or longer than $\frac{3}{4}$.

Teacher So, is $\frac{7}{8}$ greater, less, or equal to that of $\frac{3}{4}$?

Students Greater.

Teacher What's the comparison?

Students $\frac{7}{8}$ is greater than $\frac{3}{4}$.

Teacher That's right! $\frac{7}{8}$ is greater than $\frac{3}{4}$. Let's say that together.

Students $\frac{7}{8}$ is greater than $\frac{3}{4}$.

Teacher Let's write the correct symbol. Should we write the greater than symbol, less than symbol, or equal sign?

Students Greater than.

Teacher Let's write the symbol between the two numbers.
(Write.)

Teacher What does it mean to compare numbers?

Students To determine greater than, less than, or equal to.

Teacher How did we compare numbers in this example?

Students We compared each fraction using a number line and then determined whether one number was greater than, less than, or equal to the other number.

D. Problems for Use During Instruction

[See Module 2 Problem Sets.](#)

E. Vocabulary Cards for Use During Instruction

[See Module 2 Vocabulary Cards.](#)

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Module 2: Comparison

Problem Sets

- A. $>$, $<$, $=$ for numbers less than 20 (30)
- B. $>$, $<$, $=$ for numbers from 20-1,500 (30)
- C. $>$, $<$, $=$ for fractions with like denominators (15)
- D. $>$, $<$, $=$ for fractions with unlike denominators (15)
- E. $>$, $<$, $=$ for decimals to thousandths (15)

A.

6

5

A.

3

15

A.

13

3

A.

9

14

A.

2

11

A.

16

9

A.

5

13

A.

20

12

A.

7

4

A.

18

5

A.

6

16

A.

2

12

A.

14

14

A.

12

7

A.

8

19

A.

8

8

A.

10

20

A.

5

6

A.

9

9

A.

1

18

A.

15

6

A.

5

15

A.

12

12

A.

18

11

A.

7

17

A.

19

9

A.

4

4

A.

12

10

A.

14

13

A.

7

15

B.

545

534

B.

344 423

B.

287

287

B.

674

676

B.

882

828

B.

582

633

B.

656

562

B.

633

633

B.

535

553

B.

644

624

B.

448

484

B.

599

595

B.

737

735

B.

23

123

B.

45

45

B.

346

364

B.

870

807

B.

1,305

1,543

B.

128

112

B.

894 904

B.

1,321

1,321

B.

332 32

B.

65 98

B.

45

24

B.

39 29

B.

140 410

B.

285

345

B.

167

16

B.

145

45

B.

770

770

c.

$$\frac{9}{4}$$

$$\frac{8}{4}$$

c.

$$\frac{7}{2}$$

$$\frac{6}{2}$$

c.

$$\frac{3}{2}$$

$$\frac{4}{2}$$

c.

$$\frac{6}{8}$$

$$\frac{2}{8}$$

c.

$$\frac{1}{2}$$

$$\frac{1}{2}$$

c.

$$\frac{2}{6}$$

$$\frac{5}{6}$$

c.

$$\frac{11}{5}$$

$$\frac{2}{5}$$

c.

$$\frac{8}{2}$$

$$\frac{2}{2}$$

c.

$$\frac{2}{3}$$

$$\frac{10}{3}$$

c.

$$\frac{5}{4}$$

$$\frac{7}{4}$$

c.

$$\frac{8}{6}$$

$$\frac{4}{6}$$

c.

$$\frac{2}{5}$$

$$\frac{2}{5}$$

c.

$$\frac{11}{3}$$

$$\frac{7}{3}$$

c.

$$\frac{8}{5}$$

$$\frac{11}{5}$$

c.

$$\frac{10}{4}$$

$$\frac{3}{4}$$

D.

$$\frac{1}{10}$$

$$\frac{1}{2}$$

D.

$$\frac{1}{7}$$

$$\frac{1}{4}$$

D.

$$\frac{2}{6}$$

$$\frac{2}{8}$$

D.

$$\frac{1}{9}$$

$$\frac{1}{4}$$

D.

$$\frac{4}{6}$$

$$\frac{2}{8}$$

D.

$$\frac{2}{5}$$

$$\frac{4}{10}$$

D.

$$\frac{2}{3}$$

$$\frac{1}{4}$$

D.

$$\frac{4}{6}$$

$$\frac{9}{12}$$

D.

$$\frac{2}{8}$$

$$\frac{1}{2}$$

D.

$$\frac{4}{10}$$

$$\frac{1}{3}$$

D.

$$\frac{4}{5}$$

$$\frac{7}{8}$$

D.

$$\frac{1}{8}$$

$$\frac{2}{6}$$

D.

$$\frac{7}{12}$$

$$\frac{1}{5}$$

D.

$$\frac{5}{10}$$

$$\frac{1}{2}$$

D.

$$\frac{8}{12}$$

$$\frac{9}{10}$$

E.

5.6 5.2

E.

0.13

0.132

E.

0.899

0.889

E.

2.40

2.04

E.

104.5

150.4

E.

3.67

3.59

E.

0.657

0.756

E.

0.82

0.81

E.

1.906

1.903

E.

76.5

79.8

E.

5.60

5.06

E.

14.9

13.8

E.

405.4

540.4

E.

0.145

0.141

E.

1.29

1.32

Module 2: **Comparison**

Vocabulary Cards

compare
denominator
digit
equal
equal sign
equivalent
fraction
greater than
hundreds

less than
number line
numerator
ones
place value
rational number
tens
thousands

compare

To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.

$$61 > 8 \quad 37 < 80 \quad 3 = 3$$

greater than less than equal to

denominator

The term in a fraction that tells the number of equal parts in a whole.

$$2 / 3 \quad \frac{2}{3} \quad \text{In these fractions, } 3 \text{ is the denominator.}$$

digit

A symbol used to show numbers.

0 1 2 3 4 5 6 7 8 9

equal

When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.

$$3 = 3$$

equal to

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$12 + 8 = 20$$

= is the **equal sign**

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \quad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \quad \frac{10}{12} \quad \frac{8}{3}$$

greater than

When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.

$$61 > 8$$

greater than

hundreds

The digit representing 100.

less than

When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.

37 < **80**

less than

number line

A straight line with numbers placed at equal intervals along its length.



numerator

The term in a fraction that tells how many parts of a fraction.

$$2 / 3$$

$$\frac{2}{3}$$

In these fractions, **2** is the numerator.

ones

The digit representing 1.

place value

The value of a digit depending on its place in a number.

thousands	hundreds	tens	ones	.	tenths	hundredths	thousandths
8	7	6	5	.	4	3	2

rational number

Any number that can be written as a fraction.

$$\frac{3}{6} \quad \frac{10}{12} \quad \frac{8}{3}$$

tens

The digit representing 10.

thousands

The digit representing 1,000.
