



Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 3-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

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Instructional Routines for Mathematics Intervention

MODULE 3



Module 3: Representing Fractions

Mathematics Routines

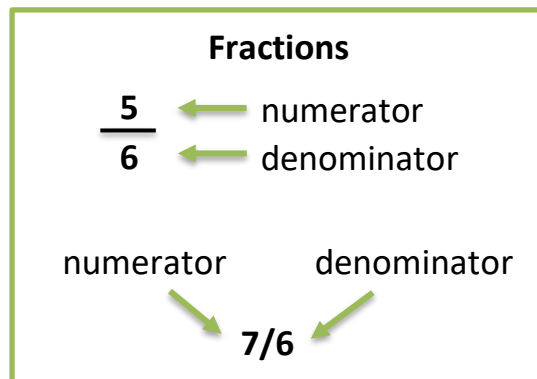
A. Important Vocabulary with Definitions

Term	Definition
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
fraction	A number representing part of a whole or set.
improper fraction	Any fraction in which the numerator is greater than or equal to the denominator.
mixed number	A whole number and a fraction combined.
numerator	The term in a fraction that tells how many parts of a fraction.
proper fraction	A fraction where the numerator is less than the denominator.

B. Background Information

In this module, we focus on representing fractions. We use three models of fractions: (1) length model, (2) area model, and (3) set model.

When referring to fractions, be sure to use proper vocabulary. Also, present fractions in different ways.



C. Routines and Examples

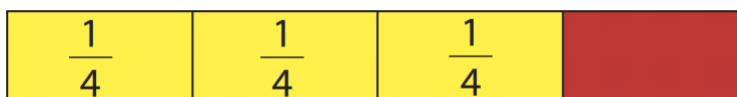
(1) Length Model

Routine

Materials:

- [Module 3 Problem Sets](#)
- [Module 3 Vocabulary Cards](#)
 - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like fraction tiles, Cuisenaire rods, or number lines

ROUTINE WITH FRACTION TILES



- Teacher** Let's show different fractions. What's a fraction?
- Students** A fraction is a number with a numerator and denominator.
- Teacher** A fraction is a number – just like 2 is a number or 13 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
- Students** A numerator and denominator.
- Teacher** The denominator tells us about the equal parts in the whole. What does the denominator tell us?
- Students** The equal parts in the whole.
- Teacher** And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
- Students** How many equal parts we show for a specific fraction.
- Teacher** So, let's show different fractions. We'll use these fraction tiles first.
(Show manipulatives.)
- Teacher** When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show?
- Students** The whole.
- Teacher** I'll place the whole in the middle of my workspace.
(Show whole.)
- Teacher** Let's show this fraction.
(Show fraction.)
- Teacher** What fraction?
- Students** ___.
- Teacher** What's the denominator of the fraction?
- Students** ___.
- Teacher** That means we want to divide this whole into ___ (denominator) equal parts. How many equal parts?

Students ___.

Teacher So, how could we divide this whole into ___ (denominator) equal parts?

Students Divide the whole into ___ equal parts with ___ fraction tiles.
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)

Teacher We showed the whole divided into ___ equal parts. Now, what's the numerator of the fraction?

Students ___.

Teacher That means we want to show ___ (numerator) of the equal parts in order to show the fraction ___. How many equal parts do we want to show?

Students ___.

Teacher Let's show ___ of the ___ equal parts.

(Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.)

Teacher What fraction did we show?

Students ___.

Teacher We showed the fraction ___. First, we divided the whole into ___ equal parts. Then, we showed the ___ equal parts of the fraction. How did we show this fraction?

Students We divided the whole into equal parts. Then, we showed the number of equal parts of the fraction.

Teacher Now, let's think about this fraction. We should interpret the left side of the whole tile as zero – like zero on a number line. What number?

Students Zero.

Teacher And if we have only one whole, we should interpret the right side of the whole tile as one – like one on a number line. What number?

Students One.

Teacher Remember to think about the whole as a number line from 0 to 1 (then 1 to 2, then 2 to 3...). That helps us learn the value of the fraction ___. Where would the fraction $\frac{1}{2}$ be on this whole?

Students In the middle between 0 and 1.

Teacher Think about $\frac{1}{2}$. What do you notice about ___ (fraction) compared to $\frac{1}{2}$?

Students ___ is greater/less than $\frac{1}{2}$.

Teacher Yes, I see ___ is greater/less than $\frac{1}{2}$. What else do you notice about ___ (fraction)?

Students ___ is greater/less than ___ (benchmark fraction).

Teacher Great work! Using these fraction tiles helps you understand the value of different fractions. Let's review. What's a denominator?

Students The equal parts in the whole.

Teacher What's a numerator?

Students How many equal parts we show for a specific fraction.

Teacher How can you use the fraction tiles to show a fraction?

Students First, you show the whole. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the equal parts in the numerator.

ROUTINE WITH CUISENAIRE RODS



Teacher Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number – just like 4 is a number or 65 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. We'll use Cuisenaire rods.
(Show manipulatives.)
Teacher When we show fractions with the Cuisenaire rods, let's first start by thinking about the whole. What should we focus on?
Students The whole.
Teacher Let's show this fraction.
(Show fraction.)
Teacher What fraction?
Students ____.
Teacher What's the denominator of the fraction?
Students ____.
Teacher That means we want to show a whole with ____ (denominator) equal parts. How many equal parts?
Students ____.
Teacher So, how could we show a whole with ____ (denominator) equal parts?
Students Show ____ (denominator) equal parts. Then, find a Cuisenaire rod with a whole that's the same length as the ____ equal parts.
Teacher Let's first find ____ (denominator) equal parts. We'll line those up and find a whole that's the same length as the ____ (denominator) equal parts.
(Show whole divided into equal parts. Place equal parts over the whole or above the whole.)
Teacher So, this rod (say color) shows the denominator. What does this rod show?

Students The denominator.

Teacher **Now, what's the numerator of the fraction?**

Students ___.

Teacher **That means we want to show ___ (numerator) of the equal parts in order to show the fraction ___. How many equal parts do we want to show?**

Students ___.

Teacher **Let's show ___ of the ___ equal parts.**
(Show the equal parts of the numerator. Leave equal parts over the whole or above the whole.)

Teacher **What fraction did we show?**

Students ___.

Teacher **We showed the fraction ___. First, we divided the whole into ___ equal parts. Then, we showed the ___ equal parts of the fraction. How did we show this fraction?**

Students We divided the whole into equal parts. Then, we showed the equal parts of the fraction.

Teacher **Now, let's think about this fraction. We should interpret the left side of the whole as zero – like zero on a number line. What number?**

Students Zero.

Teacher **And if we have only one whole, we should interpret the right side of the whole as one – like one of a number line. What number?**

Students One.

Teacher **Remember to think about the whole as a number line from 0 to 1 (then 1 to 2, then 2 to 3...). That helps us learn the value of the fraction ___. Where would the fraction $\frac{1}{2}$ be on this whole?**

Students In the middle between 0 and 1.

Teacher **Think about $\frac{1}{2}$. What do you notice about ___ (fraction) compared to $\frac{1}{2}$?**

Students ___ is greater/less than $\frac{1}{2}$.

Teacher **Yes, I see ___ is greater/less than $\frac{1}{2}$. What else do you notice about ___ (fraction)?**

Students ___ is greater/less than ___ (benchmark fraction).

Teacher **Great work! Using these Cuisenaire rods helps you understand the value of different fractions. Let's review. What's a denominator?**

Students The equal parts in the whole.

Teacher **What's a numerator?**

Students How many equal parts we show for a specific fraction.

Teacher **How can you use the Cuisenaire rods to show a fraction?**

Students First, you determine the equal parts of the whole and find a whole of the same length. Then, you look at the denominator and divide the whole into equal parts. Then, you show the numerator of the fraction by showing the number of equal parts by the numerator.

ROUTINE WITH NUMBER LINE



- Teacher** Let's show different fractions. What's a fraction?
Students A fraction is a number with a numerator and denominator.
Teacher A fraction is a number – just like 5 is a number or 17 is a number. Except with a fraction, the number has a numerator and denominator. What does a fraction have?
Students A numerator and denominator.
Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?
Students The equal parts in the whole.
Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
Students How many equal parts we show for a specific fraction.
Teacher So, let's show different fractions. Today, let's draw fractions with a number line.
(Draw a number line.)
Teacher When we show fractions with a number line, let's draw a 0 and 1 on the number line.
(Draw 0 and 1.)
Teacher Now, let's start by thinking about the whole. What should we focus on?
Students The whole.
Teacher Let's show this fraction.
(Show fraction.)
Teacher What fraction?
Students ____.
Teacher What's the denominator of the fraction?
Students ____.
Teacher That means we want to show a whole with ____ (denominator) equal parts. How many equal parts?
Students ____.
Teacher So, how could we show a whole with ____ (denominator) equal parts?
Students Draw ____ (denominator) equal parts.
Teacher Let's divide this number line into ____ (denominator) equal parts.
(Divide number line into equal parts by drawing tick marks.)
Teacher So, our number line shows ____ equal parts or the denominator. What does the number line show?
Students The denominator.
Teacher Now, what's the numerator of the fraction?
Students ____.

Teacher That means we want to show ___ (numerator) of the equal parts in order to show the fraction ___. How many equal parts do we want to show?

Students ___.

Teacher Let's draw where the fraction ___ is on the number line. I count ___ equal parts. (Draw the fraction on the number line.)

Teacher What fraction did we show?

Students ___.

Teacher We showed the fraction ___. First, we drew a number line. Then, we divided the whole into ___ equal parts. Then, we showed the fraction. How did we show this fraction?

Students We divided the number line into equal parts. Then, we determine the numerator – or the number of equal parts – and drew the fraction on the number line.

Teacher Let's think about this fraction on a number line from 0 to 1 (then 1 to 2, then 2 to 3...). That helps us learn the value of the fraction ___. Where would the fraction $\frac{1}{2}$ be on this whole?

Students In the middle between 0 and 1.

Teacher Think about $\frac{1}{2}$. What do you notice about ___ (fraction) compared to $\frac{1}{2}$?

Students ___ is greater/less than $\frac{1}{2}$.

Teacher Yes, I see ___ is greater/less than $\frac{1}{2}$. What else do you notice about ___ (fraction)?

Students ___ is greater/less than ___ (benchmark fraction).

Teacher Great work! Using the number line helps you understand the value of different fractions. Let's review. What's a denominator?

Students The equal parts in the whole.

Teacher What's a numerator?

Students How many equal parts we show for a specific fraction.

Teacher How can you use the number line to show a fraction?

Students First, draw the equal parts of the whole. Then, draw the fraction on the number line.

Example

$$\frac{3}{4}$$

EXAMPLE WITH FRACTION TILES



- Teacher** Let's show different fractions. What's a fraction?
- Students** A fraction is a number with a numerator and denominator.
- Teacher** A fraction is a number, except with a fraction, the number has a numerator and denominator. What does a fraction have?
- Students** A numerator and denominator.
- Teacher** The denominator tells us about the equal parts in the whole. What does the denominator tell us?
- Students** The equal parts in the whole.
- Teacher** And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?
- Students** How many equal parts we show for a specific fraction.
- Teacher** So, let's show different fractions. We'll use these fraction tiles first. (Show manipulatives.)
- Teacher** When we show fractions with the fraction tiles, let's first start by showing the whole. What should we show?
- Students** The whole.
- Teacher** I'll place the whole in the middle of my workspace. (Show whole.)
- Teacher** Let's show this fraction. (Show fraction.)
- Teacher** What fraction?
- Students** $\frac{3}{4}$.
- Teacher** What's the denominator of the fraction?
- Students** 4.
- Teacher** That means we want to divide this whole into 4 equal parts. How many equal parts?
- Students** 4.
- Teacher** So, how could we divide this whole into 4 equal parts?
- Students** Divide the whole into 4 equal parts with 4 one-fourth fraction tiles. (Show whole divided into equal parts. Place equal parts over the whole or above the whole.)
- Teacher** We showed the whole divided into 4 equal parts. Each equal part is one-fourth of the whole. Now, what's the numerator of the fraction?
- Students** 3.

Teacher A fraction is a number with a numerator and denominator. What does a fraction have?

Students A numerator and denominator.

Teacher The denominator tells us about the equal parts in the whole. What does the denominator tell us?

Students The equal parts in the whole.

Teacher And the numerator tells us how many equal parts should be shown in a specific fraction. What does the numerator tell us?

Students How many equal parts we show for a specific fraction.

Teacher So, let's show different fractions. We'll use these two-color counters.
(Show manipulatives.)

Teacher When we show fractions with the two-color counters, let's first start by showing the whole. What should we show?

Students The whole.

Teacher Let's show this fraction.
(Show fraction.)

Teacher What fraction?

Students $\frac{3}{4}$.

Teacher What's the denominator of the fraction?

Students 4.

Teacher That means we want to show 4 equal parts. How many equal parts?

Students 4.

Teacher So, how could we show a set with 4 equal parts?

Students Use 4 counters.

Teacher With two-color counters, we can create a set of 4 counters. We'll use the yellow side to show the denominator.
(Show 4 yellow counters in a set. Place the counters so they are touching one another.)

Teacher We showed a set divided into 4 equal parts. Now, what's the numerator of the fraction?

Students 3.

Teacher That means we want to show 3 of the equal parts in order to show the fraction $\frac{3}{4}$. How many equal parts do we want to show?

Students 3.

Teacher We can show the numerator by turning over 3 counters to the red side.
(Turn 3 counters to red side.)

Teacher What fraction did we show?

Students $\frac{3}{4}$.

Teacher We showed the fraction $\frac{3}{4}$. First, we showed a set divided into 4 equal parts. Then, we showed the 3 equal parts of the fraction. How did we show this fraction?

Students We showed a set of 4 yellow counters for the denominator, then we turned over 3 counters to the red side to show the numerator.

Teacher Now, let's think about this fraction. What would be $\frac{1}{2}$ of this set?

Students Half of the counters – or 2 counters.

Teacher What do you notice about $\frac{3}{4}$ compared to $\frac{1}{2}$?

Students $\frac{3}{4}$ is greater than $\frac{1}{2}$.

Teacher Yes, I see $\frac{3}{4}$ is greater than $\frac{1}{2}$. What else do you notice about $\frac{3}{4}$?

Students $\frac{3}{4}$ is less than 1.

Teacher Great work! Using the two-color counters helps you understand how sets can be used to show different fractions. Let's review. What's a denominator?

Students The equal parts in the whole.

Teacher What's a numerator?

Students How many equal parts we show for a specific fraction.

Teacher How can you use the two-color counters to show a fraction?

Students First, you show a set of the whole by showing a set with yellow counters. Then, you turn over counters to the red side for the numerator.

D. Problems for Use During Instruction

[See Module 3 Problem Sets.](#)

E. Vocabulary Cards for Use During Instruction

[See Module 3 Vocabulary Cards.](#)

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Module 3: Representing Fractions

Problem Sets

- A. [Proper fractions \(20\)](#)
- B. [Improper fractions \(20\)](#)
- C. [Mixed numbers \(20\)](#)

A.

$$\frac{6}{10}$$

A.

$$\frac{3}{6}$$

A.

$$\frac{1}{4}$$

A.

$$\frac{1}{12}$$

A.

$$\frac{2}{5}$$

A.

$$\frac{3}{8}$$

A.

$$\frac{4}{10}$$

A.

$$\frac{2}{12}$$

A.

$$\frac{7}{8}$$

A.

$$\frac{5}{6}$$

A.

$$\frac{3}{5}$$

A.

$$\frac{1}{6}$$

A.

$$\frac{4}{12}$$

A.

$$\frac{3}{10}$$

A.

$$\frac{2}{10}$$

A.

$$\frac{3}{9}$$

A.

$$\frac{1}{12}$$

A.

$$\frac{3}{8}$$

A.

$$\frac{9}{12}$$

B.

$$\frac{7}{5}$$

B.

$$\frac{9}{8}$$

B.

$$\frac{8}{6}$$

B.

$$\frac{5}{4}$$

B.

$$\frac{13}{10}$$

B.

$$\frac{8}{5}$$

B.

$$\frac{9}{6}$$

B.

$$\frac{6}{2}$$

B.

$$\frac{13}{12}$$

B.

$$\frac{11}{10}$$

B.

$$\frac{5}{3}$$

B.

$$\frac{9}{2}$$

B.

$$\frac{7}{3}$$

B.

$$\frac{6}{5}$$

B.

$$\frac{11}{4}$$

B.

$$\frac{9}{5}$$

B.

$$\frac{8}{6}$$

B.

$$\frac{12}{8}$$

B.

$$\frac{13}{5}$$

B.

$$\frac{15}{10}$$

c.

$$4\frac{3}{12}$$

c.

$$2\frac{3}{5}$$

c.

$$3\frac{5}{6}$$

c.

$$4\frac{1}{5}$$

c.

$$3 \frac{5}{12}$$

c.

$$1\frac{4}{5}$$

c.

$$1 \frac{3}{4}$$

c.

$$7\frac{5}{6}$$

c.

$$2\frac{2}{4}$$

c.

$$3\frac{5}{8}$$

c.

$$1 \frac{7}{8}$$

c.

$$4\frac{1}{4}$$

c.

$$7\frac{1}{2}$$

c.

$$1 \frac{5}{6}$$

c.

$$5\frac{1}{2}$$

c.

$$3\frac{4}{10}$$

c.

$$2\frac{5}{12}$$

c.

$$3\frac{1}{2}$$

c.

$$6\frac{5}{8}$$

c.

$$4 \frac{5}{10}$$

Module 3: Representing Fractions

Vocabulary Cards

denominator

equivalent

fraction

improper fraction

mixed number

numerator

proper fraction

denominator

The term in a fraction that tells the number of equal parts in a whole.

$$2 / 3 \quad \frac{2}{3} \quad \text{In these fractions, } 3 \text{ is the denominator.}$$

equivalent

Two numbers that have the same value.

$$\frac{1}{4} = \frac{2}{8} \quad \frac{2}{3} = \frac{8}{12}$$

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \quad \frac{10}{12} \quad \frac{8}{3}$$

improper fraction

Any fraction in which the numerator is greater than or equal to the denominator.

$$\frac{9}{4} \quad \frac{17}{12} \quad \frac{10}{3}$$

mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \quad 4\frac{5}{12} \quad 12\frac{4}{3}$$

numerator

The term in a fraction that tells how many parts of a fraction.

$$2/3 \quad \frac{2}{3} \quad \text{In these fractions, } 2 \text{ is the numerator.}$$

proper fraction

A fraction where the numerator is less than the denominator.

$$\frac{3}{4}$$

$$\frac{5}{6}$$

$$\frac{8}{21}$$
